Lecture 7

8. MAJOR THEOREMS OF CIRCUIT THEORY

8.1. The Superposition Theorem

The superposition theorem can be formulated as follows.

The response of a linear electric circuit to an arbitrary stimulus representing a linear combination of simpler stimuli is equal to a linear combination of responses caused by each individual stimulus.

Proof:

According to the loop current method the loop current I_{mi} of the i-th loop in the general case is defined by expression (4.73)

$$I_{mi} = \frac{\Delta_i}{\Delta},$$

where Δ, Δ_i — the determinant of the loop impedance matrix system and the determinant obtained by replacing the i-th column with the matrix-column of the loop EMF — MLE respectively. Expending the determinant Δ_i in terms of the matrix-column MLE, we get

$$I_{mi} = \frac{\Delta_{1i}}{\Delta}\dot{E}_{ml} + \frac{\Delta_{2i}}{\Delta}\dot{E}_{ml} + \ldots + \frac{\Delta_{Ni}}{\Delta}\dot{E}_{mN} = \sum_{j=1}^{N}\left(\frac{\Delta_{ji}}{\Delta}\dot{E}_{mj}\right), \quad (8.1)$$

where Δ_{1i}, Δ_{2i}, ..., Δ_{Ni}, Δ_{ji} are algebraic adjuncts of the determinant Δ to the elements Z_{1i}, Z_{2i}, ..., Z_{Ni}, Z_{ji}.

Herewith in the general case

$$\Delta_{ji} = (-1)^{j+i} M_{ji}. \tag{8.2}$$

Here M_{ji} is the minor of the determinant Δ, obtained by deletion the *j*-th row and the *i*-th column from Δ.

We can see from (8.1) that the loop current I_{mi}, considered as a circuit response, is equal to the sum of *N* components each of which is an individual current, considered as a response to the loop EMF $\dot{E}_{m\mathrm{I}}$, $\dot{E}_{m\mathrm{II}}$, ..., $\dot{E}_{mN}$, considered as individual stimuli.

Similarly, according to the node voltage method the node voltage $\dot{U}_{mi}$ of the *i*-th node in the general case is determined by the expression:

$$\dot{U}_{mi} = \frac{\Delta_i}{\Delta},$$

where Δ, Δ_i — the determinant of the matrix node admittance system and the determinant obtained from Δ by replacing the *i*-th column with the matrix-column of node currents MNC respectively.

Expending the determinant Δ_i in terms of the matrix-column MNC, we get

$$\dot{U}_{mi} = \frac{\Delta_{1i}}{\Delta}\dot{J}_{m\mathrm{I}} + \frac{\Delta_{2i}}{\Delta}\dot{J}_{m\mathrm{II}} + \ldots + \frac{\Delta_{Ni}}{\Delta}\dot{J}_{mN} = \sum_{j=1}^{N}\left(\frac{\Delta_{ji}}{\Delta}\dot{J}_{mj}\right), \tag{8.3}$$

where Δ_{1i}, Δ_{2i}, ..., Δ_{Ni}, Δ_{ji} are algebraic adjuncts of the determinant Δ key to the elements $\dot{Y}_{1i}$, $\dot{Y}_{2i}$, ..., $\dot{Y}_{Ni}$, $\dot{Y}_{ji}$. Δ_{ji} determined by (8.2).

We can see from (8.3) that a single voltage $\dot{U}_{mi}$, considered as a circuit response, is the sum of *N* components each of which represents an individual voltage, considered as a response to the node currents $\dot{J}_{m\mathrm{I}}$, $\dot{J}_{m\mathrm{II}}$, ..., $\dot{J}_{mN}$ considered as individual stimuli.

The superposition theorem does not apply to calculation of powers in terms of currents or voltages since power is a quadratic, i.e. nonlinear, function of current or voltage.

To illustrate the superposition theorem, we will apply it to calculate the circuit shown in Fig. 8.1. Here the circuit has two sources of energy (two stimuli) – a current source $\dot{J}_m$ and a voltage source $\dot{E}_m$.

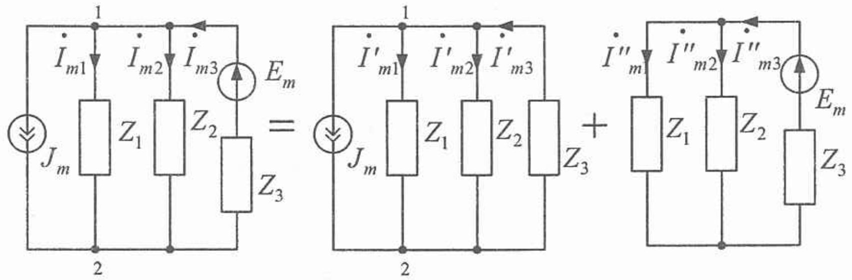


Fig. 8.1

The original circuit, according to the superposition principle, is represented by two partial circuits in the first of which there is only one source of energy — the current source $\dot{J}_m$, and in the second — the other source of energy — the voltage source $\dot{E}_m$.

Under the action of the current source $\dot{J}_m$, in the first circuit currents $\dot{I}'_{1m}$, $\dot{I}'_{2m}$, $\dot{I}'_{3m}$ flow in the branches with the resistances.

Under the action of the voltage source $\dot{E}_m$, in the second circuit currents $\dot{I}''_{1m}$, $\dot{I}''_{2m}$, $\dot{I}''_{3m}$ flow in these branches.

The total currents $\dot{I}'_{1m}$, $\dot{I}'_{2m}$, $\dot{I}'_{3m}$, proceeding from the superposition principle, are defined as the sums:

$$\begin{aligned} \dot{I}_{1m} &= \dot{I}'_{1m} + \dot{I}''_{1m}; \\ \dot{I}_{2m} &= \dot{I}'_{2m} + \dot{I}''_{2m}; \\ \dot{I}_{3m} &= \dot{I}'_{3m} + \dot{I}''_{3m}. \end{aligned} \tag{8.4}$$

Calculate the first partial circuit. The currents in the branches are determined by the alien resistance rule:

$$\dot{I}'_{1m} = \frac{\dot{J}_m \dfrac{Z_2 Z_3}{Z_2 + Z_3}}{Z_1 \dfrac{Z_2 Z_3}{Z_2 + Z_3}} = \frac{\dot{J}_m Z_2 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3};$$

$$\dot{I}'_{2m} = \frac{-\dot{J}_m \dfrac{Z_1 Z_3}{Z_1 + Z_3}}{Z_1 \dfrac{Z_1 Z_3}{Z_1 + Z_3}} = \frac{-\dot{J}_m Z_1 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3};$$

$$\dot{I}'_{3m} = \frac{\dot{J}_m \dfrac{Z_1 Z_2}{Z_1 + Z_2}}{Z_1 \dfrac{Z_1 Z_2}{Z_1 + Z_2}} = \frac{\dot{J}_m Z_1 Z_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}.$$

Calculating the second partial circuit, we get:

$$\dot{I}''_{3m} = \frac{\dot{E}_m}{Z_3 + \dfrac{Z_1 Z_2}{Z_1 + Z_2}} = \frac{\dot{E}_m (Z_2 + Z_3)}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3};$$

$$\dot{I}''_{1m} = \frac{\dot{I}''_{3m} Z_2}{Z_1 + Z_2} = \frac{\dot{E}_m Z_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3};$$

$$\dot{I}''_{2m} = \frac{\dot{I}''_{3m} Z_1}{Z_1 + Z_2} = \frac{\dot{E}_m Z_1}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}.$$

The total currents of (8.4):

$$\dot{I}_{1m} = \frac{(\dot{E}_m - \dot{J}_m Z_3) Z_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3};$$

$$\dot{I}_{2m} = \frac{(\dot{E}_m - \dot{J}_m Z_3) Z_1}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}; \qquad (8.5)$$

$$\dot{I}_{3m} = \frac{\dot{E}_m (Z_1 + Z_2) + \dot{J}_m Z_1 Z_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}.$$

8.2. The Equivalent Generator Theorem

This theorem is also known as Thevenin-Norton's theorem and it consists of two parts.

Thevenin's theorem — the equivalent voltage source theorem — can be formulated as follows.

The current in any branch of a linear electric circuit does not change if the rest of the circuit is replaced by its equivalent voltage source the EMF of which is equal to the voltage at the terminals of an open branch, and the internal resistance – to the resistance between the discontinuity points.

Proof:

Consider an electric circuit with voltage sources $\dot{E}_{m1}$, $\dot{E}_{m2}$, ..., $\dot{E}_{mn}$ and impedances Z_1, Z_2, ..., Z_n (Fig. 8.2, *a*). In this circuit, select the branch with an impedance Z_{id}, connected between terminals *k, l,* with a current $\dot{I}_m$.

Include into the *k-l* branch two voltage sources that are oppositely directed but whose EMF are equal in magnitude $E'_m = E''_m = \dot{U}_{mxx}$, where $\dot{U}_{mxx}$ is the voltage between the open terminals *k, l* (Fig. 8.2, *b*).

It is obviously that the current $\dot{I}_m$ in the circuit will not change. On the basis of the superposition theorem, this circuit can be represented as the sum of the two circuits, in the first of which the EMFs are $\dot{E}_{m1}$, $\dot{E}_{m2}$, ..., $\dot{E}_{mn}$ and E'_m and the direction of the EMF E'_m is opposite to the direction of the current $\dot{I}_m$ (Fig. 8.2, *c*) and in the second — there is only E''_m (Fig. 8.2, *d*).

Then the current is

$$\dot{I}_m = \dot{I}'_m + \dot{I}''_m.$$

It is obvious that the current $\dot{I}_m$ in the circuit of fig. 8.2, *c* is equal to zero as the equivalent EMF of the sources $\dot{E}_{m1}$, $\dot{E}_{m2}$, ..., $\dot{E}_{mn}$ and E'_m are equal but oppositely directed. Therefore the current is

$$\dot{I}_m = \dot{I}''_m.$$

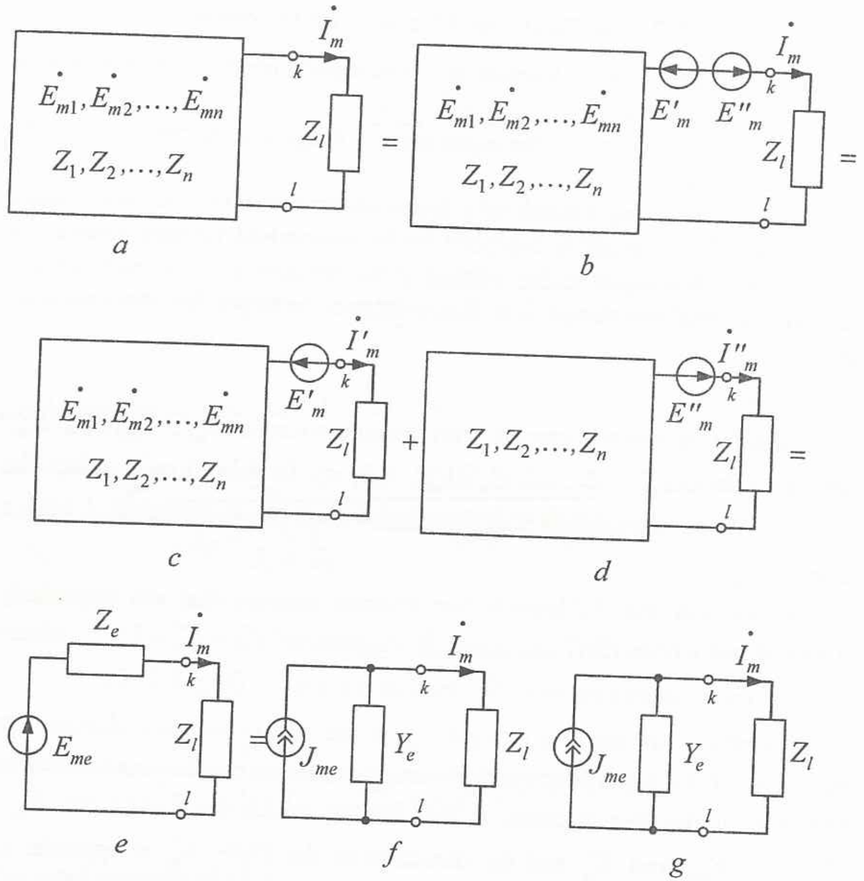


Fig. 8.2

If we replace the impedances Z_1, Z_2, ... , Z_n by the equivalent impedance Z_{eq}, then we have for Fig. 8.2, *d*:

$$\dot{I}_m = \dot{I}''_m = \frac{\dot{E}''_m}{Z_i + Z_{eq}} = \frac{\dot{E}_{meq}}{Z_i + Z_{eq}} = \frac{\dot{E}_{meq}}{Z_i + Z_{eq}}.$$

The equivalent circuit with Z_{eq} and $\dot{E}_{meq}$ is shown in Fig. 8.2, *e*. We can see that when the *k*–*l* branch of this circuit is broken, we get the open-circuit voltage at these terminals:

$$\dot{U}_{mxx} = \dot{E}_{meq}.$$

The theorem is proved.

Norton's theorem — the equivalent current source theorem — can be formulated as follows.

The current in any branch of a linear electric circuits does not change if the rest of the circuit is replaced by its equivalent current source the current of which is equal to the short-circuit current of this branch, and the internal conductance — to the conductance between the discontinuity points of this branch.

The proof of this theorem follows immediately from Fig. 8.2, *e* if the equivalent voltage source with the EMF $\dot{E}_{meq}$ and the internal impedance Z_{eq} is replaced by the equivalent current source $\dot{J}_{meq}$ and the internal conductance Y_{eq} (Fig. 8.2, *g*).

Here

$$\dot{J}_{meq} = \frac{\dot{E}_{meq}}{Z_{eq}}; \; Y_{eq} = \frac{1}{Z_{eq}}.$$

We can see from Fig. 8.2, *g* that for the short-circuited *k*, *l* terminals the current is

$$\dot{I}_{msc} = \frac{\dot{E}_{meq}}{Z_{eq}} = \dot{J}_{meq}.$$

We can also see from Fig. 8.2, *g*, that when the terminals *k*, *l* are open, the conductance from the side of the open terminals is

$$Y_{k1} = \frac{1}{Z_{eq}} = Y_{eq}.$$

The theorem is proved.

To illustrate the equivalent generator theorem, we will apply this theorem to determine the current $\dot{I}_{1m}$ in the network of Fig. 8.1.

We will break the branch with Z_1, with the current $\dot{I}_{1m}$ flowing through it, to calculate the voltage $\dot{U}_{mid12}$ between the terminals 1, 2 of the circuit (Fig. 8.3). Transform the voltage source with the EMF $\dot{E}_m$ and the internal resistance Z_3 into its equivalent current source

$$\dot{J}_{m3} = \frac{\dot{E}_m}{\dot{Z}_3}.$$

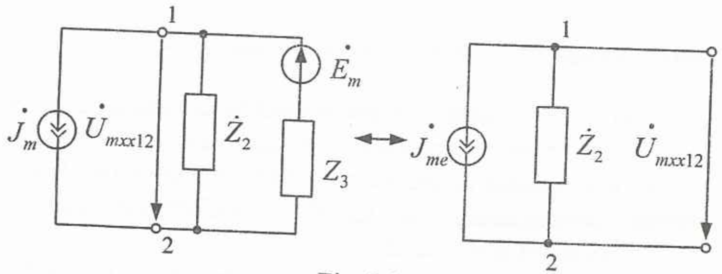


Fig. 8.3

Then, by means of algebraic summation of the current sources $\dot J_m$ and $\dot J_{m3}$, determine the equivalent current source:

$$\dot J_{meq} = \dot J_m - \dot J_{m3} = \dot J_m - \frac{\dot E_m}{Z_3}. \tag{8.6}$$

The internal admittance Y_e:

$$Y_{eq} = \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{Z_2 + Z_3}{Z_2 Z_3}. \tag{8.7}$$

Then, from (8.6) and (8.7) we get

$$\dot U_{mid12} = \frac{-\dot J_{meq}}{Y_{eq}} = -\left(\dot J_m - \frac{\dot E_m}{Z_3}\right)\frac{Z_2 Z_3}{Z_2 + Z_3} = \frac{(\dot E_m - J_m Z_3) Z_2}{Z_2 + Z_3}. \tag{8.8}$$

Now from (8.6)–(8.8) we get

$$\dot I_{1m} = \frac{\dot U_{mid12}}{Z_1 + \frac{1}{Z_e}} = \frac{(\dot E_m - J_m Z_3)}{Z_2 + Z_3} \cdot \frac{1}{Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}} = \frac{(\dot E_m - J_m Z_3) Z_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3},$$

which coincides with (8.5).

8.3. The Reciprocity Theorem

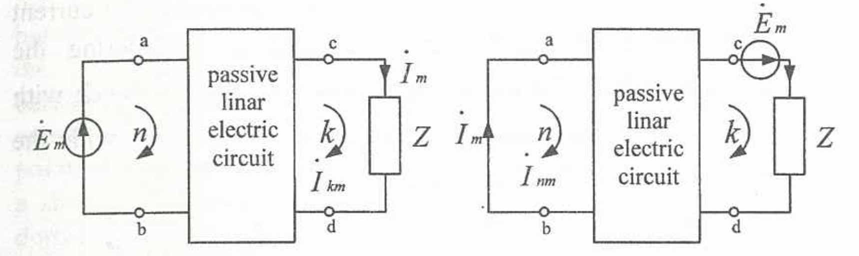
The reciprocity theorem can be formulated as follows.

If a voltage source with an EMF $\dot E_m$ or a current source $\dot J_m$ is included in the branch *a–b* of a linear electric circuit, that does not contain other energy sources, and creates a current $\dot I_m$ in the branch *c–d*,

then the same voltage source $\dot E_m$ or current source $\dot J_m$, if included in the branch *c-d*, creates the same current $\dot I_m$ in the branch *a–b*.

Proof:

Consider the circuits shown in Fig. 8.4. Here the voltage source $\dot E_m$, included in the branch *a–b* of the passive linear electric circuit, creates a current $\dot I_m$ in the branch *c–d* with an impedance Z (Fig. 8.4, *a*). Put this source to the branch *c–d*. Determine the current in the branch *a–b*. Let the branch *a–b* be included in the loop *n*, and the branch *c–d* — in the loop *k* of the linear electric circuit. Calculate the circuit according to the loop current method.



a *b*

Fig. 8.4

1. Let the voltage source $\dot E_m$ be included in the loop *n* (Fig. 8.4, *a*). Then the current in the *k*-th loop is:

$$\dot I_{km} = \dot I_m = \frac{\Delta_k}{\Delta} = \frac{\Delta_{nk}}{\Delta}\dot E_m,$$

where Δ — the determinant of the loop impedance matrix system; Δ_k — the determinant derived from Δ by replacing its *k*-th column with the loop EMF column; Δ_{nk} — the determinant obtained by expanding Δ_k in terms of the loop EMF column.

As the EMF $\dot E_m$ is included only in the loop *n*, and the rest of the circuit is passive, then all the determinants except Δ_{nk} are equal to zero.

2. Let the voltage source $\dot E_m$ be included in the loop *k* (Fig. 8.4, *b*).

Then the current through the *n*-th loop:

$$\dot I_{nm} = \dot I_n = \frac{\Delta_n}{\Delta} = \frac{\Delta_{kn}}{\Delta}\dot E_m,$$

where Δ_{kn} — the determinant, obtained by expanding Δ_n in terms of the loop EMF column. As the EMF $\dot{E}_m$ is included only in the loop k, and the rest of the circuit is passive, then all the determinants except Δ_{kn} are equal to zero.

It is known, that a matrix of loop impedances is symmetric about the main diagonal, that is $\Delta_{nk} = \Delta_{kn}$. Therefore

$$\dot{I}_{km} = \dot{I}_{nm} = \dot{I}_m .$$

The theorem is proved.

To illustrate the reciprocity theorem, we will determine the current $\dot{I}''_{1m}$ in the circuit of Fig. 8.1 that we used in considering the superposition principle. Put the voltage source $\dot{E}_m$ to the branch with the impedance Z_1. Take the direction of $\dot{E}_m$ coinciding with the direction of the current $\dot{I}''_{1m}$. Then the current is

$$\dot{I}''_{m3} = \frac{\dot{I}''_{1m} Z_2}{Z_2 + Z_3} = \frac{\dot{E}_m Z_2}{\left(Z_1 + \dfrac{Z_2 Z_3}{Z_2 + Z_3} \right)} = \frac{\dot{E}_m Z_2}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3} = \dot{I}''_{1m},$$

which coincides with the result obtained in 8.1.

The reciprocity theorem holds for linear passive circuits only. For nonlinear and active circuits, the reciprocity theorem does not hold in the general case.

8.4. The Compensation Theorem

The compensation theorem can be formulated as follows. The currents and voltages in an electric circuit will not change, if any branch of this circuit is replaced by an ideal voltage source, whose EMF is equal to the voltage at the terminals of this branch and oppositely directed to it, or if it is replaced by an ideal current source, whose current is equal to the current of the branch and coincides with it in direction.

Proof:

Let us point out the branch *c-d* with an impedance Z and current $\dot{I}_m$ in the electric circuit shown in Fig. 8.5, *a*.

The branch voltage is $\dot{U}_{cdm} = \dot{I}_m Z$.

Include in the branch *c–d* two voltage sources whose EMF $\dot{E}_m$ are equal to the voltage $\dot{U}_{cdm}$ and oppositely directed (Fig. 8.5, *b*). Obviously, the current $\dot{I}_m$ will not change. As the voltages between the points *a–b* and *b–d* are equal in magnitude but opposite in direction, the resultant voltage between the points *a–d* is equal to zero, and these points can be connected by a short-circuit jumper (the dotted line in Fig. 8.5, *b*). The result is the equivalent circuit in Fig. 8.5, *c*, in which the branch *c-d* includes only the EMF $\dot{E}_m = \dot{U}_{cdm}$ and the current in which is equal to the original current $\dot{I}_m$.

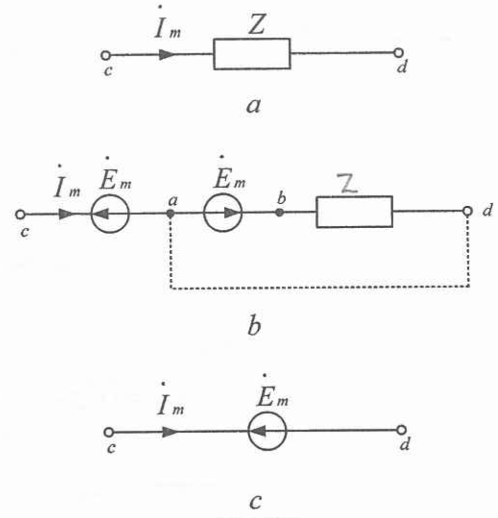


Fig. 8.5

8.5. Tellegen's theorem

Tellegen's theorem is extremely versatile. It applies to any electric circuit with lumped parameters, which contains any kinds of elements: linear and non-linear, passive and active, time-varying or constant. This versatility results from the fact that Tellegen's theorem is based on Kirchhoff's laws. Tellegen's theorem (published in 1952) can be formulated as follows: The sum of the voltages and currents in each branch of an electric circuit is equal to zero. That is

$$\sum_K u_k i_k = 0,$$

where u_k, i_k — instantaneous values of voltage and current in the k-th branch of the circuit.

Proof.

Let us consider. The nodes α and β of the electric circuit shown in Fig. 8.6 are connected by the branch k with an impedance Z, through which flows the current $i_{\alpha\beta}$ of the indicated direction.

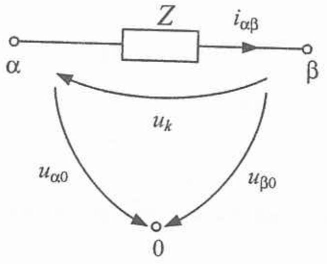


Fig. 8.6

The node voltages relative to the basic node 0 are denoted by $u_{\alpha 0}$ and $u_{\beta 0}$. The voltage of the k-th branch is denoted by u_k.

We will also denote

$$i_k = i_{\alpha\beta}.$$

It is obvious that

$$u_k = u_{\alpha 0} - u_{\beta 0}.$$

We will write the product

$$u_k i_k = \left(u_{\alpha 0} - u_{\beta 0}\right) i_{\alpha\beta} \tag{8.9}$$

or

$$u_k i_k = \left(u_{\beta 0} - u_{\alpha 0}\right) i_{\beta\alpha}, \tag{8.10}$$

where

$$i_{\beta\alpha} = -i_{\alpha\beta}.$$

Adding (8.9) to (8.10) we get

$$u_k i_k = 0{,}5\left[(u_{\alpha 0} - u_{\beta 0}) i_{\alpha\beta} + (u_{\beta 0} - u_{\alpha 0}) i_{\beta\alpha}\right]. \tag{8.11}$$

Sum up (8.11) for all branches of the electric circuit. We get for b-branches

$$\sum_{k=1}^{b} u_k i_k = 0{,}5 \sum_{\alpha=0}^{n} \sum_{\beta=0}^{n} \left(u_{\alpha 0} - u_{\beta 0}\right) i_{\alpha\beta}. \tag{8.12}$$

Here the double summation is introduced as the multiplication within the summation symbol is performed for all possible combinations of branches. Here n – the number of nodes of the circuit. If there is no branch connecting the node α with the node β, then we write $i_{\alpha\beta} = i_{\beta\alpha}$ in (8.12). The right part of equation (8.12) can be divided as follows:

$$0{,}5 \sum_{\alpha=0}^{n} \sum_{\beta=0}^{n} \left(u_{\alpha 0} - u_{\beta 0}\right) i_{\alpha\beta} = 0{,}5\left(\sum_{\alpha=0}^{n} \sum_{\beta=0}^{n} u_{\alpha 0 \alpha\beta} - \sum_{\alpha=0}^{n} \sum_{\beta=0}^{n} u_{\beta 0} i_{\alpha\beta}\right) =$$
$$= 0{,}5\left[\sum_{\alpha=0}^{n} u_{\alpha 0}\left(\sum_{\beta=0}^{n} i_{\alpha\beta}\right) - \sum_{\beta=0}^{n} u_{\beta 0}\left(\sum_{\alpha=0}^{n} i_{\alpha\beta}\right)\right]. \tag{8.13}$$

In (8.13) for each value α the sum is

$$\sum_{\beta=0}^{n} i_{\alpha\beta} = 0 \tag{8.14}$$

since it is the sum of the currents in all branches proceeding from the node α.

Also, for each value β the sum is

$$\sum_{\alpha=0}^{n} i_{\alpha\beta} = 0 \tag{8.15}$$

since it is the sum of the currents in all branches proceeding from the node β. It follows from Kirchhoff's current law.

Thus, from (8.12)–(8.15) we get

$$\sum_{k=1}^{b} u_k i_k = 0. \tag{8.16}$$

The left part of expression (8.16) is the sum of the instantaneous powers of all branches of the circuit, which is identical to the instantaneous power balance condition (4.23). Hence, the power balance condition is a special case of Tellegen's theorem. The theorem is proved.

To illustrate Tellegen's theorem, consider the electric circuit shown in Fig. 8.7.

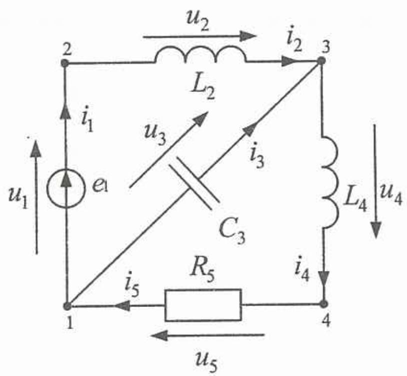


Fig. 8.7

The instantaneous currents and voltages for each branch are specified here. Assume that for a certain instant of time:

$$i_1 = 1\text{ A},\ i_2 = 1\text{ A},\ i_3 = -3\text{ A},\ i_4 = -2\text{ A},\ i_5 = -2\text{ A};$$
$$u_1 = 1\text{ B},\ u_2 = 2\text{ B},\ u_3 = 1\text{ B},\ u_4 = 4\text{ B},\ i_5 = -5\text{ B}.$$

Tellegen's theorem holds for electric circuits for which Kirchhoff's laws are observed. Here according to Kirchhoff's current law:

for node 1: $-i_1 - i_3 + i_5 = -1 - (-3) - 2 = 0;$

for node 2: $i_1 - i_2 = 1 - 1 = 0;$

for node 3: $i_2 + i_3 - i_4 = 1 - 3 - (-2) = 0;$

for node 4: $i_4 - i_5 = -2 - (-2) = 0.$

According to Kirchhoff's voltage law:

for the loop e_1, L_2, C_3: $-u_1+u_2-u_3=-1+2-1=0$;
for the loop C_3, L_4, R_5: $u_3+u_4+u_5=1+4-5=0$.

Now, by Thellegen's theorem

$$\sum_{k=0}^{5} u_k i_1 = -u_1 i_1 + u_2 i_2 + u_3 i_3 + u_4 i_4 + u_5 i_5 =$$
$$= -1\cdot 1 + 2\cdot 1 + 1(-3) + 4(-2) + (-5)(-2) = 0.$$

Example 1

Determine the current in the circuit (Fig. 8.8, *a*) using the superposition theorem. The parameters of the circuit elements are: $R_1=6\ \Omega$; $R_2=4\ \Omega$; $R_3=12\ \Omega$; $E_1=120$ V; $E_2=100$ V.

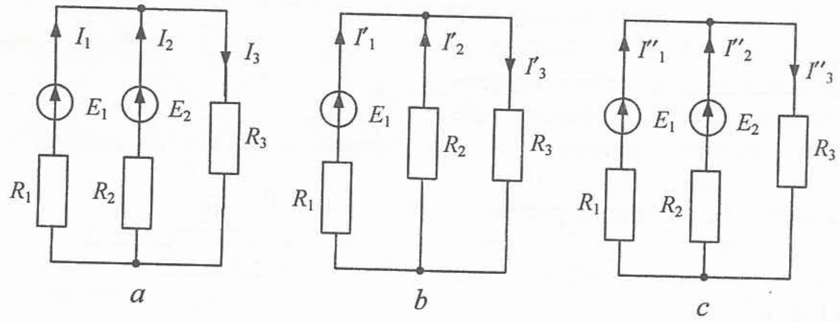


Fig. 8.8

Solution

In accordance with the superposition theorem, the current I_3 can be found as the sum of the partial currents I_3' and I_3'', flowing in the same branch under the action of the sources E_1 and E_2 separately. The circuits in which the currents I_3' and I_3'' are to be determined are presented in Fig. 8.8, *b*, *c*:

$$I_3' = \frac{E_1}{R_1 + \dfrac{R_2 R_3}{R_2+R_3}} \cdot \frac{R_2}{R_2+R_3} = \frac{120}{6+\dfrac{4\cdot 12}{4+12}} \cdot \frac{4}{4+12} = 3{,}33\ \text{A};$$

$$I_3'' = \frac{E_2}{R_2 + \dfrac{R_1 R_3}{R_1+R_3}} \cdot \frac{R_1}{R_1+R_3} = \frac{100}{4+\dfrac{6\cdot 12}{6+12}} \cdot \frac{6}{6+12} = 4{,}17\ \text{A};$$

$$I_3 = I_3' + I_3'' = 3{,}33 + 4{,}17 = 7{,}5\ \text{A}.$$